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Modulational instability of ion-acoustic waves in a plasma with negative ions

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A nonlinear Schrödinger equation that governs the nonlinear interaction of a quasistatic plasma slow response with ion-acoustic waves for a warm-ion, hot-electron plasma composed of positive ions, electrons, and negative ions is derived. It is found that the negative-ion species render these waves unstable in a wave-number (k) range that is perfectly stable in the absence of negative-ion species. It is also found that for a given value of negative-ion concentration α, such that $0<\alpha<\alpha_c$, there exists an upper bound on k, i.e., k_{ch}, below which the waves would be modulationally unstable. However, when $\alpha>\alpha_c$, there exists a lower bound on k, k_{cl}, above which the waves are modulationally unstable. The variation of the wave-number range over which the ion-acoustic wave is unstable with the negative-ion concentration, charge-multiplicity ratio, and relative mass of the two ion species is discussed. The predictions of the theory are found to be in substantial agreement with experimental observations of modulational instability.

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I. INTRODUCTION

During the past few years, the nonlinear slow quasistatic plasma response to ion-acoustic waves leading to modulation of ion-acoustic waves has been studied by several authors [1–5]. These studies show that the ion-acoustic waves are modulationally stable for parallel modulation, whereas in the case of oblique modulation, the waves become modulationally unstable [4,5]. In the present paper we intend to investigate the modulational instability of ion-acoustic waves in a plasma composed of positive ions, electrons, and negative ions as a result of nonlinear slow quasistatic plasma response.

In recent years there has been considerable interest in understanding the behavior of plasmas containing positive ions, electrons, and a significant concentration of negative ions. With recent technology it is possible to produce plasmas in which the negative-ion density is much larger than the electron density. In such plasmas, electrons make no appreciable contribution to many characteristic plasma phenomena, being accompanied by a lack of electron shielding of potential variation, and new features of plasma phenomena are expected to appear. Negative-ion plasmas are found in the D region of the ionosphere [6], plasma processing reactors [7], and neutral-beam sources [8]. They have been studied in high-current-density diffuse discharges which are used to pump high-power gaseous lasers [9]. Dusty plasmas also contain massive negatively charged particles; therefore, studies of negative-ion plasmas may also be applied in attempting to understand the properties of this type of plasma.

Laboratory experiments in plasmas containing negative ions have investigated ion-acoustic waves [10,11], electrostatic ion cyclotron waves [12], solitons [13,14], beam-plasma interactions [15], turbulence [16], double layers [17], and self-modulation of ion-acoustic waves [18]. Sheehan and Ryhn [19] have presented a useful summary of negative-ion plasma sources. Negative ions are produced by electron attachment to neutral particles when an electronegative gas, i.e., the gas with a large electron-attachment cross section, e.g., halogens and hexafluorides, is introduced into electric gas discharges. Wong, Mamas, and Arnush [10] have described a method for producing plasmas with nearly all electrons replaced by negative ions. In their experiment they made use of SF_6, a gas of great electron affinity especially for $T_e \leq 0.2$ eV, and obtained a negative-ion plasma with $1-\alpha \leq 10^{-3}$ at a neutral gas pressure of 5×10^{-4} Torr, where $\alpha = n_- / n_+ \simeq 1 - n_e / n_+$. Later, Hershkowitz and Intrator [20] also obtained a negative-ion plasma with $1-\alpha \leq 10^{-3}$ at a neutral pressure less than 1×10^{-4} Torr. Recently, Sato [21] reported that it is possible to obtain negative-ion plasmas containing K^+ and SF_6^- ions, in which the electron fraction becomes as small as $1-\alpha \simeq 10^{-4}$ at a SF_6 gas pressure

$$P(SF_6) \simeq 5\times10^{-4}\ \text{Torr}\ .$$

Considering the harmonic-generated nonlinearities, the modulational instability of ion-acoustic waves in a plasma consisting of singly ionized positive and negative ions, along with electrons, has been studied by Saito, Watanabe, and Tanaca [23]. They found that above a critical wave number k_c, the wave becomes modulationally unstable and the value of k_c strongly depends on the negative-ion concentration. They have also found that at a critical concentration of negative ions, the critical wave number k_c reduces to zero and the wave of any wave number becomes modulationally unstable. Tsukabayashi and Nakamura [18] have reported experimental observation of modulational instability of large-amplitude ion-acoustic waves in a plasma consisting of Ar^+ and F^- ions and electrons.

The object of the present paper is to study the modulational stability of ion-acoustic waves in a plasma consisting of positive ions, electrons, and negative ions. We have investigated the effect of negative-ion concentration, charge-multiplicity ratio, and relative mass of the two-ion species on the instability of modulated ion-acoustic waves due to nonlinear interaction with slow quasistatic plasma response. The presence of negative-ion species changes the coefficient of the nonlinear term in the nonlinear Schrödinger equation. It is found that in the presence of negative-ion species, the sign of the coefficient of the nonlinear term (i.e., Q) depends on the negative-ion concentration α, charge-multiplicity ratio (ϵ_z), and relative mass of the two-ion species. Hence, the unstable wave-number range strongly depends on α, ϵ_z, and μ. Our theoretical results are in fairly good agreement with the experimental observation of Tsukabayashi and Nakamura [18].

Basic equations are given in Sec. II. In Sec. III we derive the nonlinear Schrödinger equation. Section IV contains a discussion, and conclusions are presented in Sec. V.

II. BASIC EQUATIONS

We consider an ion-acoustic wave traveling in the x direction in a collisionless plasma consisting of warm two-ion species (i.e., positive ions and negative ions) and hot isothermal electrons. The nonlinear interaction of finite-amplitude ion-acoustic waves with the background collisionless plasma is governed by the following set of normalized fluid equations:

$$\frac{\partial n_{i1}}{\partial t}+\frac{\partial}{\partial x}(n_{i1}V_{i1})=0 , \tag{1}$$

$$\frac{\partial V_{i1}}{\partial t}+V_{i1}\frac{\partial V_{i1}}{\partial x}=-\frac{1}{\beta}\frac{\partial \phi}{\partial x}-\frac{1}{\beta Z_1}\frac{T_i}{T_e}\frac{1}{n_{i1}}\frac{\partial n_{i1}}{\partial x} , \tag{2}$$

$$\frac{\partial n_{i2}}{\partial t}+\frac{\partial}{\partial x}(n_{i2}V_{i2})=0 , \tag{3}$$

$$\frac{\partial V_{i2}}{\partial t}+V_{i2}\frac{\partial V_{i2}}{\partial x}=\left[\frac{\mu\epsilon_z}{\beta}\right]\frac{\partial \phi}{\partial x}-\left[\frac{\mu}{\beta Z_1}\right]\frac{T_i}{T_e}\frac{1}{n_{i2}}\frac{\partial n_{i2}}{\partial x} , \tag{4}$$

$$\frac{\partial \phi}{\partial x}=\frac{1}{n_e}\frac{\partial n_e}{\partial x} , \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-\frac{n_{i1}}{(1-\alpha\epsilon_z)}+\frac{\alpha\epsilon_z}{(1-\alpha\epsilon_z)}n_{i2} ,$$
$$\beta=\frac{(1+\alpha\mu\epsilon_z^2)}{(1-\alpha\epsilon_z)},\quad \epsilon_z=\frac{Z_2}{Z_1},\quad \mu=\frac{M_1}{M_2},\ \text{and}\ \alpha=\frac{n_{i2}^{(0)}}{n_{i1}^{(0)}} . \tag{6}$$

In the above equations, n_{i1}, V_{i1} and n_{i2}, V_{i2} are the density and fluid velocity of the positive ion and the negative ion, respectively. n_e is the electron density, ϕ is the electrostatic potential, μ is the mass ratio of the positive ions to the negative ions, α is the density ratio of negative ions to positive ions, and ϵ_z is the charge-multiplicity ratio of negative ions to positive ions. In Eq. (5), we have neglected the electron inertia. The quantities V, ϕ, t, and x are normalized with respect to ion-acoustic wave speed in the mixture $C_s=(T_e\beta Z_1/M_1)^{1/2}$; thermal potential (T_e/e); inverse of the ion plasma frequency in the mixture ω_{pi}^{-1}, where

$$\omega_{pi}=(4\pi n_e^{(0)}e^2Z_1\beta/M_1)^{1/2} ;$$

and Debye length λ_D, respectively. Densities n_{i1}, n_{i2}, and n_e are normalized with their corresponding equilibrium densities, i.e., $n_{i1}^{(0)}$, $n_{i2}^{(0)}$, and $n_e^{(0)}$.

III. DERIVATION OF THE NONLINEAR SCHRÖDINGER EQUATION

We are interested in investigating the slow quasistatic plasma response to the ion-acoustic waves; therefore, we write the field quantities in normalized form as follows:

$$n_j=1+n_j^h+n_j^l , \tag{7}$$

$$V_j=V_j^h+V_j^l , \tag{8}$$

$$\phi=\phi^h+\phi^l , \tag{9}$$

where $n_j^{h(l)}\ll 1$. The superscripts h and l represent the corresponding quantities associated with the ion wave (high frequency) and with the quasistatic plasma slow motion (low frequency), respectively.

Using Eqs. (7)–(9) in Eq. (5), the electron-density perturbation associated with the ion-acoustic waves in the presence of the plasma slow motion is given by

$$n_e^h=(1+n_e^l)\phi^h . \tag{10}$$

Now we combine Eqs. (1) and (2), and (3) and (4); then, introducing Eqs. (6)–(9), we obtain the following nonlinear equation for the ion-acoustic waves in the presence of the plasma slow response:

$$\left[\left[1-\frac{\partial^2}{\partial x^2}\right]\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right]\phi^h+\left[\frac{\partial^2}{\partial t^2}-\frac{1}{\beta}\frac{\partial^2}{\partial x^2}\right]n_e^l\phi^h=0 . \tag{11}$$

In deriving Eq. (11), ions are assumed to be much colder than electrons, i.e., $T_i/T_e\ll 1$. In addition, we have used the quasineutral and quasistatic behavior of a plasma towards slow response, i.e.,

$$\left[\frac{n_{i1}^l}{(1-\alpha\epsilon_z)}-\frac{\alpha\epsilon_z}{(1-\alpha\epsilon_z)}n_{i2}^l\right]=n_e^l\ \text{and}\ V_i^l=V_e^l\simeq 0 .$$

In the absence of nonlinear interaction, linearization of (11) yields the following dispersion relation:

$$\omega^2=\frac{k^2}{(1+k^2)} , \tag{12}$$

where k is the wave vector of the ion-acoustic wave. The modulation group velocity (i.e., the velocity with which the modulation propagates) of the wave is given by

$$V_g=\frac{d\omega}{dk}=\frac{\omega^3}{k^3}=\frac{1}{(1+k^2)^{3/2}} . \tag{13}$$

Now we calculate the electron-density perturbation n_e^l associated with the quasistatic plasma slow motion. Taking the momentum balance equations for ions and electrons, i.e., Eqs. (2), (4), and (5), using Eqs. (7)–(9), and then averaging over the ion-acoustic wave period, we get

$$\frac{1}{2}\frac{\partial}{\partial x}\langle|(V_{i1}^h)^2|\rangle=-\frac{1}{\beta}\frac{\partial\phi^l}{\partial x}-\frac{1}{\beta Z_1}\frac{T_i}{T_e}\frac{\partial n_{i1}^l}{\partial x}\ , \tag{14}$$

$$\frac{1}{2}\frac{\partial}{\partial x}\langle|(V_{i2}^h)^2|\rangle=\left[\frac{\mu\epsilon_z}{\beta}\right]\frac{\partial\phi^l}{\partial x}-\left[\frac{\mu}{\beta Z_1}\right]\frac{T_i}{T_e}\frac{\partial n_{i2}^l}{\partial x}\ , \tag{15}$$

$$\frac{\partial\phi^l}{\partial x}=\frac{\partial n_e^l}{\partial x}\ , \tag{16}$$

where we have assumed that the phase velocity of the modulation is much smaller than the electron and ion thermal velocities. In the above equations, angular brackets denote averaging over the ion-acoustic wave period. The left-hand sides of Eqs. (14) and (15) represent the ion ponderomotive force. Equations (14) and (15) can be written as

$$\frac{\beta Z_1}{2}\frac{\partial}{\partial x}\langle|(V_{i1}^h)^2|\rangle=-Z_1\frac{\partial\phi^l}{\partial x}-\frac{T_i}{T_e}\frac{\partial n_{i1}^l}{\partial x}\ , \tag{17}$$

$$\frac{\beta Z_1}{2\mu}\frac{\partial}{\partial x}\langle|(V_{i2}^h)^2|\rangle=Z_2\frac{\partial\phi^l}{\partial x}-\frac{T_i}{T_e}\frac{\partial n_{i2}^l}{\partial x}\ . \tag{18}$$

Now, multiplying Eq. (17) by $1/(1-\alpha\epsilon_z)$ and Eq. (18) by $\alpha\epsilon_z/(1-\alpha\epsilon_z)$, and then subtracting, we get

$$\frac{\beta Z_1}{2(1-\alpha\epsilon_z)}\frac{\partial}{\partial x}\langle|(V_{i1}^h)^2|\rangle-\frac{\beta Z_1}{2\mu}\frac{\alpha\epsilon_z}{(1-\alpha\epsilon_z)}\frac{\partial}{\partial x}\langle|(V_{i2}^h)^2|\rangle=-\frac{Z_1(1+\alpha\epsilon_z^2)}{(1-\alpha\epsilon_z)}\frac{\partial\phi^l}{\partial x}-\frac{T_i}{T_e}\frac{\partial n_e^l}{\partial x}\ , \tag{19}$$

where we have used

$$\left[\frac{n_{i1}^l}{(1-\alpha\epsilon_z)}-\frac{\alpha\epsilon_z}{(1-\alpha\epsilon_z)}n_{i2}^l\right]=n_e^l\ .$$

Using Eq. (16) in Eq. (19), we get

$$\frac{\beta Z_1}{2(1-\alpha\epsilon_z)}\frac{\partial}{\partial x}\langle|(V_{i1}^h)^2|\rangle-\frac{\beta Z_1}{2\mu}\frac{\alpha\epsilon_z}{(1-\alpha\epsilon_z)}\frac{\partial}{\partial x}\langle|(V_{i2}^h)^2|\rangle=-\left[\frac{Z_1(1+\alpha\epsilon_z^2)}{(1-\alpha\epsilon_z)}+\frac{T_i}{T_e}\right]\frac{\partial n_e^l}{\partial x}\ . \tag{20}$$

Now from the ion-momentum equations, i.e., Eqs. (2) and (4), we get

$$V_{i1}^h=\left[\frac{k}{\beta\omega}\right]\phi^h \tag{21}$$

and

$$V_{i2}^h=-\left[\frac{\mu\epsilon_z}{\beta}\right]\frac{k}{\omega}\phi^h\ . \tag{22}$$

Using Eqs. (21) and (22) in Eq. (20), we get

$$n_e^l=-\frac{Z_1(1+k^2)}{2\beta(1-\alpha\epsilon_z)}\frac{(1-\alpha\mu\epsilon_z^3)}{\left[\dfrac{Z_1(1+\alpha\epsilon_z^2)}{(1-\alpha\epsilon_z)}+\gamma\right]}|\phi^h|^2\ , \tag{23}$$

where $\gamma=T_i/T_e$ is the ratio of the ion to the electron temperature. Substituting Eq. (23) into Eq. (11), we get

$$\left[\left[1-\frac{\partial^2}{\partial x^2}\right]\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right]\phi^h-\frac{Z_1(1+k^2)}{2\beta(1-\alpha\epsilon_z)}\frac{(1-\alpha\mu\epsilon_z^3)}{\left[\dfrac{Z_1(1+\alpha\epsilon_z^2)}{(1-\alpha\epsilon_z)}+\gamma\right]}\times\left[\frac{\partial^2}{\partial t^2}-\frac{1}{\beta}\frac{\partial^2}{\partial x^2}\right]|\phi^h|^2\phi^h=0\ . \tag{24}$$

We assume that the nonlinear interaction of the quasistatic plasma slow response with the ion-acoustic waves gives rise to an envelope of waves whose amplitudes vary on time and space scales much more slowly than those of the ion-acoustic oscillations; accordingly, we let

$$\phi^h=\epsilon^{1/2}\phi^h(\xi,\tau)\exp(-i\omega t+ikx)+\text{c.c.}\ , \tag{25}$$

where ϵ indicates the magnitude of small but finite amplitude ϕ^h, ξ and τ are defined such that

$$\xi=\epsilon^{1/2}(x-V_g t)\ , \tag{26a}$$

and

$$\tau=\epsilon t\ . \tag{26b}$$

Substituting Eqs. (25) and (26) into Eq. (24), and using Eqs. (12) and (13), we get, to $O(\epsilon^{3/2})$, the following nonlinear Schrödinger equation:

$$i\frac{\partial\phi^h}{\partial\tau}+\left[-\frac{3}{2}\frac{k}{(1+k^2)^{5/2}}\right]\frac{\partial^2\phi^h}{\partial\xi^2}+\frac{Z_1k^3}{4(1+k^2)^{1/2}}\frac{1}{\left[\dfrac{Z_1(1+\alpha\epsilon_z^2)}{(1-\alpha\epsilon_z)}+\gamma\right]}\times\left[1-\frac{\alpha\mu\epsilon_z^2(1+\epsilon_z)}{(1+\alpha\mu\epsilon_z^2)}\right]\times\left[1-\frac{\alpha\epsilon_z(1+\mu\epsilon_z)}{(1+\mu\alpha\epsilon_z^2)}\frac{(1+k^2)}{k^2}\right]|\phi^h|^2\phi^h=0$$

or

$$i\frac{\partial\phi^h}{\partial\tau}+P\frac{\partial^2\phi^h}{\partial\xi^2}+Q|\phi^h|^2\phi^h=0\ , \tag{27}$$

where P and Q are the coefficients of dispersive and nonlinear terms, respectively. The dispersion term P is half the modulation group-velocity dispersion (dV_g/dk), i.e.,

$$P=\frac{1}{2}\frac{dV_g}{dk}=-\frac{3}{2}\frac{k}{(1+k^2)^{5/2}} \tag{28}$$

and

$$Q=\frac{Z_1k^3}{4(1+k^2)^{1/2}}\frac{F_2F_3}{F_1}\ , \tag{29}$$

where

$$F_1=\left[\frac{Z_1(1+\alpha\epsilon_z^2)}{(1-\alpha\epsilon_z)}+\gamma\right]\ , \tag{30a}$$

$$F_2=\left[1-\frac{\alpha\mu\epsilon_z^2(1+\epsilon_z)}{(1+\alpha\mu\epsilon_z^2)}\right]\ , \tag{30b}$$

and

$$F_3=\left[1-\frac{\alpha\epsilon_z(1+\mu\epsilon_z)}{(1+\mu\alpha\epsilon_z^2)}\frac{(1+k^2)}{k^2}\right]\ . \tag{30c}$$

It may be pointed out that our expressions for P and Q reduce to those obtained by Shukla [1] in the limit $\alpha=0$ with $\mu=1$ and $\epsilon_z=1$. It may be noted that for a given plasma with Z_1, Z_2, $n_e^{(0)}$, and $n_{i1}^{(0)}$ fixed, α is given by

$$\alpha=\frac{1}{Z_2}\left[Z_1-\frac{n_e^{(0)}}{n_{i1}^{(0)}}\right]\ .$$

It shows that only for the case of $Z_1=Z_2=1$ can α be equal to 1, but this corresponds to $n_e^{(0)}=0$. In this situation the plasma is composed of positive ions and negative ions only, and ion-acoustic waves are no longer possible. In all other cases, α is less than unity.

We notice from Eqs. (28) and (29) that the coefficient of the dispersive term P is independent of μ, ϵ_z, and α, i.e., parameters related to negative-ion species. But the coefficient of the nonlinear term Q depends on μ, ϵ_z, and α. The sign of Q depends on the functions F_2 and F_3. It will be negative if one of them (i.e., F_2 or F_3) is positive and the other is negative. The sign of F_2 changes as α passes through α_c, where α_c, the critical value of α, is given by $F_2=0$. For a given value of α such that $\alpha<\alpha_c$, F_2 remains positive, whereas for $\alpha>\alpha_c$, F_2 becomes negative. Similarly, the sign of F_3 changes as k passes through k_c, where k_c, the critical value of k, is given by $F_3=0$. For a given value of k such that for $k<k_c$, F_3 remains negative, whereas for $k>k_c$, F_3 becomes positive. Thus, we have

$$Q\begin{cases}>0 \text{ when (i) } 0<\alpha<\alpha_c \text{ and } k>k_c \\ \qquad\quad \text{(ii) } \alpha>\alpha_c \text{ and } k<k_c & \text{(31a)} \\ <0 \text{ when (i) } 0<\alpha<\alpha_c \text{ and } k<k_c \\ \qquad\quad \text{(ii) } \alpha>\alpha_c \text{ and } k>k_c\ , & \text{(31b)}\end{cases}$$

where

$$\alpha_c=1/\mu\epsilon_z^3 \tag{32a}$$

and

$$k_c=\left[\frac{\alpha\epsilon_z(1+\mu\epsilon_z)}{(1-\alpha\epsilon_z)}\right]^{1/2}\ . \tag{32b}$$

Therefore, we expect that the presence of negative-ion species (say, fluoride F^- in an argon Ar^+ plasma or sulphur hexafluorides $SF_6{}^-$ in a potassium K^+ plasma or fluoride F^- in a cesium plasma Cs^+ or oxygen O^{-2} in an argon Ar^+ plasma) would affect the stable and unstable domains significantly.

IV. DISCUSSION AND COMPARISON WITH EXPERIMENTAL RESULTS

The amplitude of the modulated ion-acoustic wave, defined by the nonlinear Schrödinger equation, i.e., Eq. (27), will be modulationally unstable when P and Q have the same sign, i.e., $PQ>O$ [22], in the modulation wave number ($\tilde{K}$) range

$$O<\tilde{K}^2<2Q|a_0|^2/P\ . \tag{33}$$

In the present case, i.e., the plasma composed of positive ions, negative ions, and electrons, Eqs. (28) and (29) show that the sign of Q depends on the functions F_2 and F_3, whereas P is always negative. Further, as discussed in Sec. III, PQ is positive or negative depending on the values of k and α. Thus, we have

$$PQ\begin{cases}>0 \text{ when (i) } \alpha<\alpha_c \text{ and } k<k_c \\ \qquad\quad \text{(ii) } \alpha>\alpha_c \text{ and } k>k_c \\ <0 \text{ when (i) } \alpha<\alpha_c \text{ and } k>k_c \\ \qquad\quad \text{(ii) } \alpha>\alpha_c \text{ and } k<k_c\ .\end{cases}$$

Thus (for $k>0$ and $\alpha>0$) the k-α space is divided into four parts, in two of which the wave is unstable (i.e., $PQ>0$), and in the other two it is stable (i.e., $PQ<0$). It may also be noted that at $k=k_c$ as well as at $\alpha=\alpha_c$, Q reduces to zero. Therefore, for these cases (i.e., $k=k_c$ and $\alpha=\alpha_c$), the growth rate will be zero, which implies that the wave will be marginally stable.

To investigate the effect of negative-ion concentration, the relative mass of two ion species, and the charge-multiplicity ratio on the modulationally unstable region in the k-α plane, we plot the curves $F_2=0$ and $F_3=0$ on a k-α plane for an argon plasma containing fluoride ions F^- as a negative-ion species (Fig. 1). The wave is unstable in regions enclosed by $F_3=0$ and $F_2=0$, denoted by I and III in Fig. 1, but is stable in the open regions whose boundaries on one side are curves $F_2=0$ or $F_3=0$ and on the other side extend to infinity in the α-k space denoted by II and IV in Fig. 1.

A comparison with the work of Saito, Watanabe, and Tanaca [23] shows that an instability exists in both cases of negative-ion plasma. However, the two theories show that the waves are modulationally unstable with a different domain of instability in different parameter ranges. The physical mechanism that Saito, Watanabe, and Tanaca have considered is the harmonic generation due to self-interaction of ion-acoustic waves, and which they have taken into account up to the second-order harmonic-generated nonlinearities. On the other hand, we have investigated the modulational instability of ion-acoustic waves due to nonlinear interaction with slow

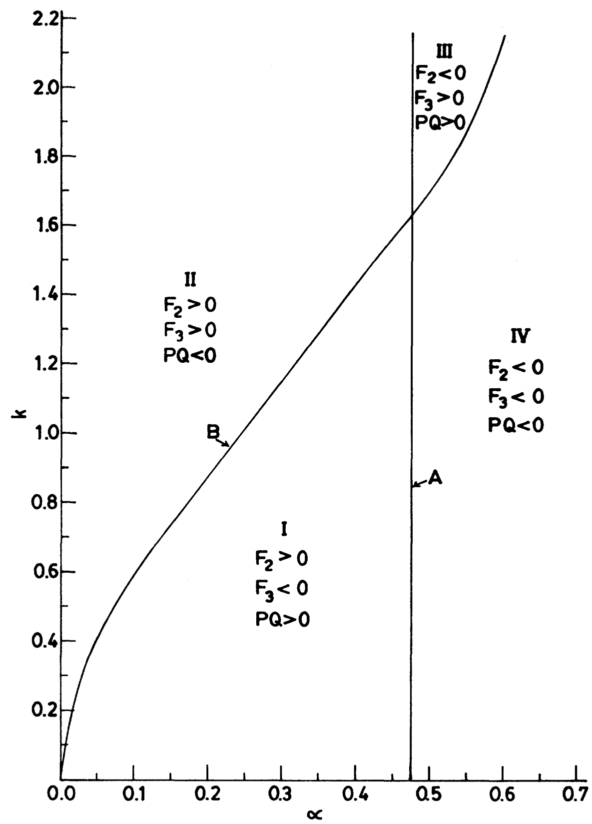

FIG. 1. Plot of $F_2=0$ and $F_3=0$ in the k-α plane for the plasma having Ar^+ and F^- ions and electrons with $Z_1=Z_2=1$, $\epsilon_z=1$, and $\mu=2.1$. The curve A refers to $F_2=0$ and B refers to $F_3=0$. P is always negative. The regions I and III represent the modulationally unstable regions for the ion-acoustic wave.

response quasistatic plasma. In this case the interaction of a finite-amplitude ion-acoustic wave with quasistatic nonresonant density fluctuation is significant. It should be noted that this slow response is driven by the resulting ponderomotive force. The crucial qualitative as well as quantitative differences between their studies and our results are as follows.

Saito, Watanabe, and Tanaca showed that for a given value of negative-ion concentration α, there always exists a lower bound on k, k_{cl}, above which the waves are modulationally unstable. They also showed that by increasing the value of α, the critical wave number k_{cl} first decreases, reduces to zero, and then increases. However, in the case of slow response quasistatic plasma, the case we have considered, it is found that for a given value of α such that $0<\alpha<\alpha_c$, there exists an upper bound on k, k_{ch}, below which the waves would be modulationally unstable, whereas when $\alpha>\alpha_c$, there exists a lower bound on k, k_{cl}, above which the waves are modulationally unstable. In our case there does not exist such a value of α at which the wave is modulationally unstable for all wave numbers. It may also be noted that for a given value of α, the wave-number range for which the wave is unstable is different. For some values of k, the wave is stable according to one but unstable according to another.

For the two-component plasma, i.e., consisting of positive ions and electrons, the modulational instability of ion-acoustic waves due to nonlinear interaction with slow response quasistatic plasma has been studied by Shukla [1]. In that case the coefficient of the dispersive term, i.e., P, remains always negative, and the coefficient of the nonlinear term, i.e., Q remains always positive, irrespective of the wave number k. Therefore, PQ remains always less than zero, i.e., $PQ<0$. He has found that the ion-acoustic waves in a plasma consisting of positive ions and electrons remain always modulationally stable. A comparison of our results with Shukla's result shows that, due to the presence of negative-ion species, the wave becomes unstable for that wave-number range for which it was perfectly stable in the absence of negative-ion species.

Tsukabayashi and Nakamura [18] have reported experimental observation of modulational instability of ion-acoustic waves in a negative-ion plasma containing Ar^+ and F^- ions and electrons. In their experiment they have taken the following plasma parameters:

$$n_e=(3-8)\times 10^8\ \mathrm{cm}^{-3}\ ,$$

$$T_e=1.2-2.0\ \mathrm{eV}\ ,$$

and

$$T_i/T_e=\tfrac{1}{15}\ .$$

The value of negative-ion concentration α is taken as $\alpha=0.1$. They found that at the externally applied pulse of frequency 340 kHz and peak-to-peak amplitude $V_{ex}=0.36$ V, the detected wave has a large amplitude with almost $\delta n/n=0.1$ and is self-modulated, keeping the symmetrical form.

It may be noted, from Eq. (23), that if the amplitude of the wave is large, in that case the magnitude of the electron-density perturbation n_e^l associated with the quasistatic plasma slow motion will also be large. In the experiment of Tsukabayashi and Nakamura, the amplitude of an ion-acoustic wave is large (the normalized perturbed number density is about 10%); therefore, the nonlinear interaction with the background plasma cannot be ignored. Hence, we think that in the above experiment, the nonlinear slow quasistatic plasma response to an ion-acoustic wave would be strong and should contribute significantly to the modulation process.

Using the experimental data and the dispersion relation, one can calculate the corresponding value of wave number k, and it lies in the wave-number range of $0.051k_D$ and $0.091k_D$. Hence, experimentally, the wave is unstable if the wave number k lies between $0.051k_D$ and $0.091k_D$. For the plasma parameters of this experiment, we have, according to our theory, the critical negative-ion concentration $\alpha_c=0.47$. The experimental value of $\alpha=0.1$ is less than α_c, and we theoretically expect instability for wave numbers lying in the range $0<k<k_{ch}$, where $k_{ch}=0.59k_D$. This range covers the experimental values, and instability is observed in the experiment. Thus the experimental observations of Tsukabayashi and Nakamura support our theory.

We have also compared another result, i.e., modulation frequency, with the experimental value. Tsukabayashi and Nakamura have reported that the experimentally observed modulation frequency lies between 40–50 kHz. Theoretically, for a given value of wave number k, the modulation frequency can be calculated using the group-velocity relation, i.e., Eq. (13), and modulation wave number $\tilde{K}$, lying in the range given by Eq. (33). According to our theory, for the given range of experimental value of wave number k, i.e., $0.051k_D$–$0.091k_D$, the calculated value of modulation frequency lies in the range 25–41 kHz. This also shows that the predicted value of modulation frequency is in reasonable agreement with the experimental value.

The above comparison shows that the predictions of the theory are in substantial agreement with the experimental observations. Hence, we think that the nonlinearity utilized in the present paper is more suitable to describe the instability observed in the experiment.

V. CONCLUSIONS

Our main conclusions are as follows.

(i) It is found that negative-ion species render ion-acoustic waves unstable in the wave-number range which is perfectly stable in the absence of negative-ion species.

(ii) It is also found that for a given value of negative-ion concentration α such that $0<\alpha<\alpha_c$, where $\alpha_c = 1/\mu\epsilon_z^3$, determined by the constituents of a plasma, there exists an upper bound on k, k_{ch}, below which the waves would be modulationally unstable. By increasing the value of α, the critical wave number k_{ch} increases; therefore, the unstable region increases.

(iii) When $\alpha > \alpha_c$, in that case there exists a lower bound on k, k_{cl}, above which the waves are modulationally unstable. On further increasing α, k_{cl} increases, decreasing the instability domain. However, for such types of plasmas for which α_c is greater than or equal to 1, i.e., $\alpha_c \geq 1$, the lower bound does not exist, because in that case $\alpha > \alpha_c$ is unphysical.

(iv) For a plasma with parameters corresponding to the experiment of Tsukabayashi and Nakamura [18], we have, according to our theory, $\alpha_c = 0.47$. Thus, for $\alpha = 0.1$, which is less than α_c, we expect instability for a wave number lying in the range $0 < k < k_{ch}$, where $k_{ch} = 0.59k_D$. Thus the wave is expected to show instability, as observed in the experiment. Experimentally observed modulation frequency lies between 40–50 kHz, whereas, according to our theory, its value lies in the range 25–41 kHz. This also shows that the predicted value is in reasonable agreement with the experimental value, which shows that the theoretical predictions are in substantial agreement with the experimental observations.

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